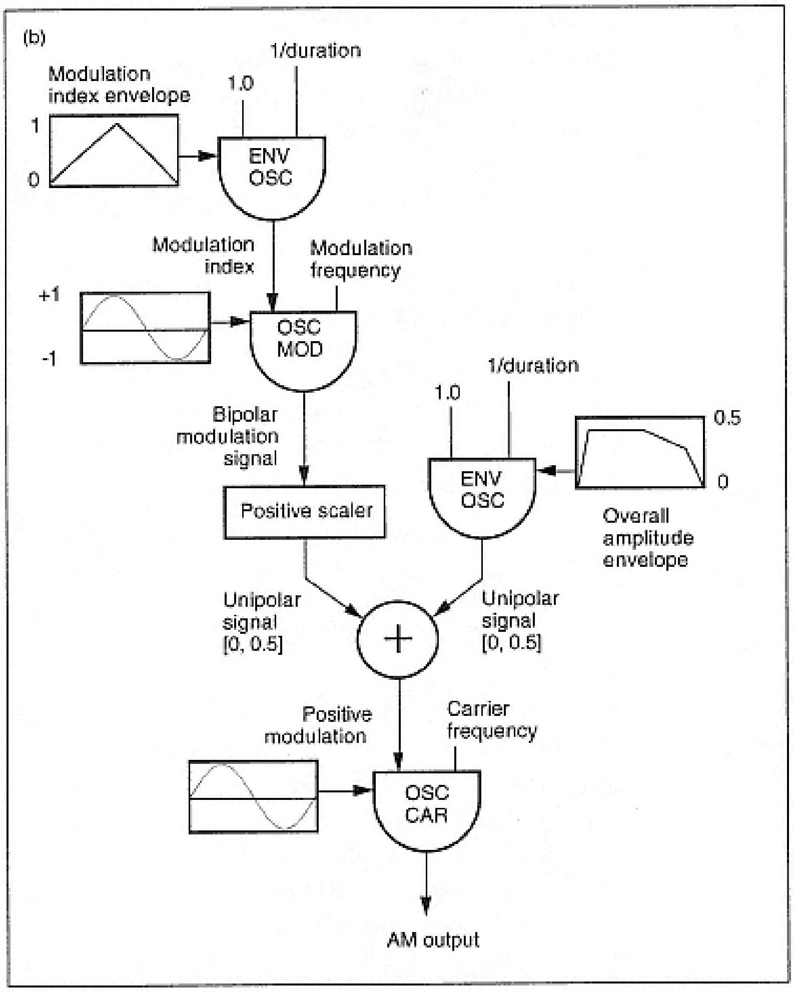

Figure 6.8 (b)

Background:
Frequency Modulation

Applications of FM in communications systems date back to the nineteenth century. The theory behind FM of radioband frequencies (in the MHz range) was established early in the twentieth century (Carson 1922; van der Pol 1930; Black 1953). These studies are worth reading today, particularly Black's book, which walks the reader through a well-planned tour of the hills and dales of waveform modulation.

John Chowning at Stanford University was the first to explore systematically the musical potential of digital FM synthesis (Chowning 1973). Prior to this, most digital sound had been produced by fixed-waveform, fixed-spectrum techniques. Time-varying additive and subtractive synthesis were rare and costly from a computational standpoint. Since most digital synthesis work had to be done on multiple-user computers, there was a strong incentive to develop more efficient techniques, with the emphasis on time-varying spectra. This motivation was explained by Chowning as follows:

> *In natural sounds the frequency components of the spectrum are dynamic, or time variant. The energy of the components often evolves in complicated ways; in particular during the attack and decay portions of the sound.*
> (Chowning 1973)

Hence, Chowning sought a way to generate synthetic sounds that had the animated spectra characteristic of natural sounds. The breakthrough came when he was experimenting with extreme vibrato techniques, where the vibrato becomes so fast it effects the timbre of the signal:

> *I found that with two simple sinusoids I could generate a whole range of complex sounds which done by other means demanded much more powerful and extensive tools. If you want to have a sound that has, say 50 harmonics, you have to have 50 oscillators. And I was using two oscillators to get something that was very similar.*
> (Chowning 1987)

After careful experiments to explore the potential of the technique, Chowning developed a patent on an implementation of FM. In 1975 the Japanese firm Nippon Gakki (Yamaha) obtained a license to apply this patent in their products. After several years of development and extensions to the basic technique (described later), Yahama introduced the expensive GS1 digital synthesizer ($16,000, housed in a wooden pianolike case) in 1980. But it was the introduction of the highly successful DX7 synthesizer ($2000) in the fall of 1983 that made FM synonymous with digital synthesis to hundreds of thousands of musicians.

Frequency Modulation and Phase Modulation

FM and the closely related technique called *phase modulation* (PM) represent two virtually identical cases of the same type of *angle modulation* (Black 1953, pp. 28 30). The amplitudes of the partials generated by the two methods exhibit slight differences, but in musical practice there is no great distinction between PM and FM, particularly in the case of time-varying spectra. Hence we will not discuss PM further in this book. (A variation called *phase distortion* is discussed later in this chapter, however.) For details on the distinction between PM and FM, see Bate (1990), Holm (1992), and Beauchamp (1992).

Simple FM

In the basic frequency modulation technique (referred to as *simple FM* or *Chowning FM*), a carrier oscillator is modulated in frequency by a modulator oscillator (Chowning 1973, 1975). Figure 6.9 diagrams a simple FM instrument. (A slight discrepancy exists between the amplitudes of the spectrum components emitted by the instrument shown in figure 6.9 and the spectra described by the classic FM formula, presented in a moment. Overall these differences are minor. For a summary see Holm 1992 and Beauchamp 1992.)

Looking at the spectrum shown in figure 6.10 we can immediately see the difference between FM and the RM and AM methods presented earlier. Instead of just one sum and one difference sideband, FM of two sinusoids generates an series of sidebands around a carrier frequency *C*. Each sideband spreads out at a distance equal to a multiple of the modulating frequency *M*. Later we investigate the number of sidebands; suffice it to say

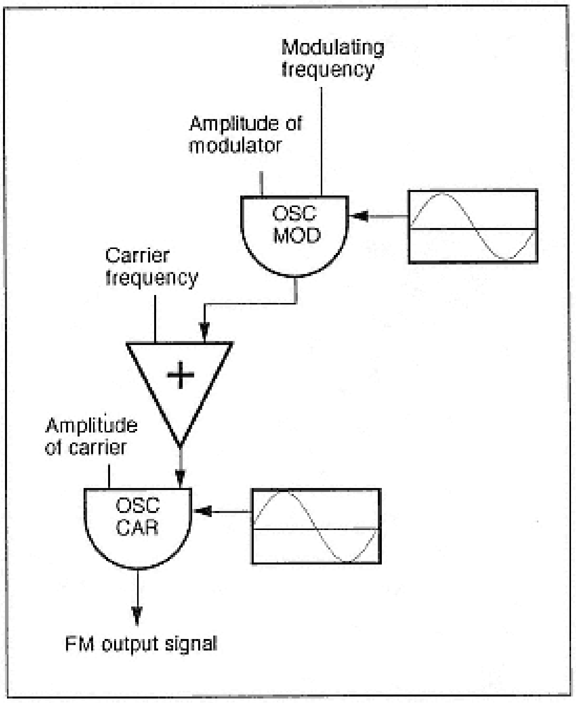

Figure 6.9
A simple FM instrument. The bipolar output of the modulating oscillator is added to the fundamental carrier frequency, causing it to vary up and down. The amplitude of the modulator determines the amount of modulation, or the frequency deviation from the fundamental carrier frequency.

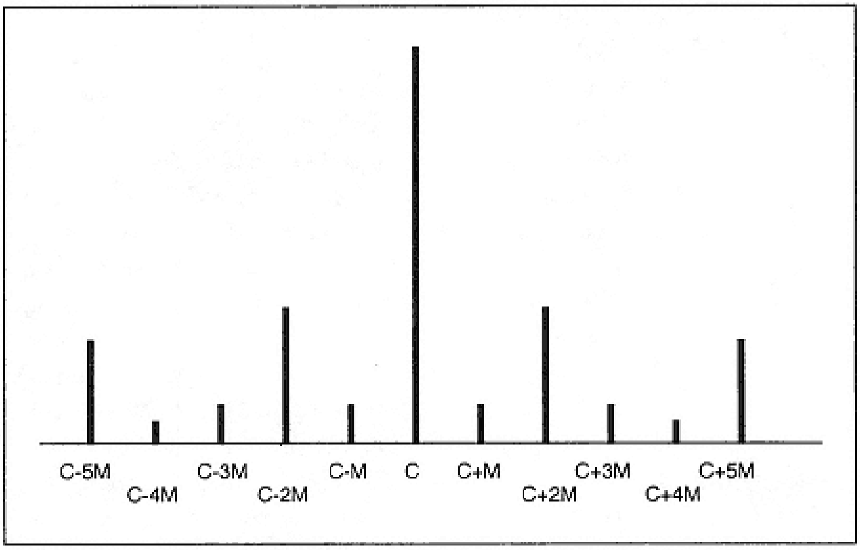

Figure 6.10
FM spectrum showing sidebands equally spaced around the carrier *C* at multiples of the modulator *M*.

now that the number of sidebands generated depends on the amount of modulation applied to the carrier.

C:M *Ratio*

The position of the frequency components generated by FM depends on the ratio of the carrier frequency to the modulating frequency. This is called the *C:M ratio*. When *C:M* is a simple integer ratio, such as 4:1 (as in the case of two signals at 800 and 200 Hz), FM generates harmonic spectra, that is, sidebands that are integer multiples of the carrier and modulating frequencies:

C = 800 Hz	(carrier)
C + *M* = 1000 Hz	(sum)
C + (2 *M*) = 1200 Hz	(sum)
C + (3 *M*) = 1400 Hz, etc.	(sum)
C *M* = 600 Hz	(difference)
C (2 *M*) = 400 Hz	(difference)
C (3 *M*) = 200 Hz, etc.	(difference)

When *C*:*M* is not a simple integer ratio, such as 8:2.1 (as in the case of two signals at 800 and 210 Hz), FM generates inharmonic spectra (noninteger multiples of the carrier and modulator):

C = 800 Hz	(carrier)
$C + M$ = 1010 Hz	(sum)
C + (2 M) = 1120 Hz	(sum)
C + (3 M) = 1230 Hz, etc.	(sum)
C M = 590 Hz	(difference)
C (2 M) = 380 Hz	(difference)
C (3 M) = 170 Hz, etc.	(difference)

Modulation Index and Bandwidth

The bandwidth of the FM spectrum (the number of sidebands) is controlled by the *modulation index* or *index of modulation I. I* is defined mathematically according to the following relation:

$I = D/M$

where D is the amount of frequency deviation (in Hz) from the carrier frequency. Hence, D is a way of expressing the *depth* or amount of the modulation. So if D is 100 Hz and the modulator M is 100 Hz, then the index of modulation is 1.0.

Figure 6.11 plots the effects of increasing the modulation index. When $I = 0$ (figure 6.11a) the frequency deviation is zero so there is no modulation. When I is greater than zero, sideband frequencies occur above and below the carrier C at intervals of the modulator M. As I increases, so does the number of sidebands. Notice in that as I increases, energy is "stolen" from the carrier and distributed among the increasing number of sidebands.

As a rule of thumb, the number of significant sideband pairs (those that are more than 1/100th the amplitude of the carrier) is approximately $I + 1$ (De Poli 1983). The total bandwidth is approximately equal to twice the sum of the frequency deviation D and the modulating frequency M (Chowning 1973). In formal terms:

FM bandwidth ~ 2 $(D + M)$.

Because the bandwidth increases as the index of moduation increases, FM can simulate an important property of instrumental tones. Namely, as the

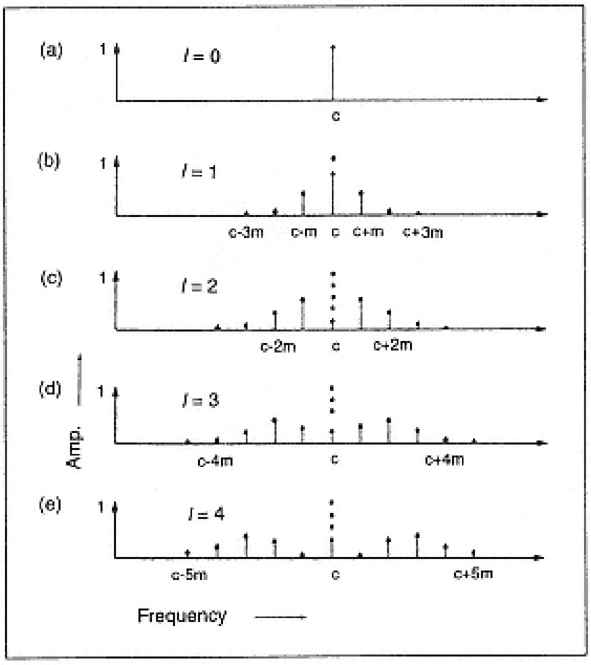

Figure 6.11
FM spectrum with increasing modulation index. (*a*) Carrier. (*b*) (*e*) Carrier plus sidebands for $I = 0$ (see *a*) to 4 (see *e*). The sidebands are spaced at intervals of the modulating frequency *M* and are symmetrical about the carrier *C*. (After Chowning 1973.)

amplitude increases, so does the bandwidth. This is typical of many instruments, such as strings, horns, and drums, and is realized in FM by using similar envelope shapes for both the carrier amplitude and index of modulation.

Reflected Sidebands

For certain values of the carrier and modulator frequencies and *I*, extreme sidebands reflect out of the upper and lower ends of the spectrum, causing audible side effects. An upper partial that is beyond the Nyquist frequency (half the sampling rate) "folds over" (aliases) and reflects back into the lower portion of the spectrum. (Chapter 1 describes foldover in more detail.)

When the lower sidebands extend below 0 Hz, they reflect back into the spectrum in 180-degree *phase-inverted form*. By "phase inverted" we mean that the waveform flips over the *x*-axis, so that the positive part of a sine wave becomes negative and the negative part becomes positive. Phase-

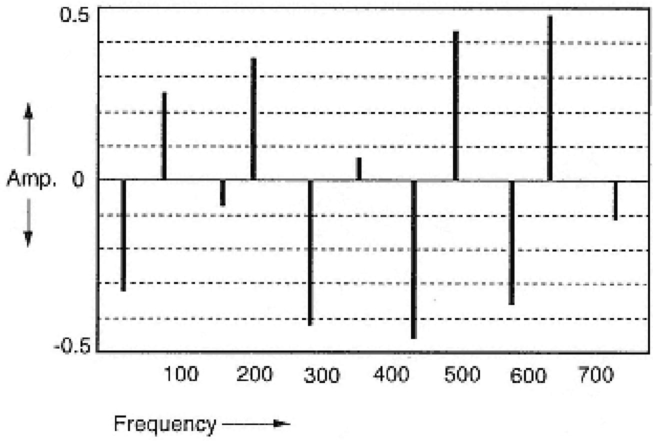

Figure 6.12
Spectral plot showing the effects of reflected low- frequency sidebands. The *C:M* ratio is $1\sqrt{2}$, and the modulation index is 5. The downward lines indicate phase-inverted reflected components.
(After Chowning 1973.)

inverted partials are drawn as lines extending downward, as in figure 6.12. In general, negative frequency components add richness to the lower-frequency portion of the spectrum, but if negative components overlap exactly with positive components, they can cancel out each other.

The FM Formula

When the carrier and the modulator are both sine waves, the formula for a frequency modulated signal *FM* at time *t* is as follows:

$$FM_t = A \quad \sin(C_t + [I \quad \sin(M_t)])$$

where *A* is the peak amplitude of the carrier, $C_t = 2\pi \quad C$, $M_t = 2\pi \quad M$, and *I* is the index of modulation. As this formula shows, simple FM is quite efficient, requiring just two multiplies, an add, and two table lookups. The table lookups reference sine waves stored in memory.

Bessel Functions

The amplitudes of the individual sideband components vary according to a class of mathematical functions called *Bessel functions of the first kind and the nth order* $J_n(I)$, where the argument to the function is the modulation index *I*. The FM equation just given can be reëxpressed in an equivalent representation (adapted from De Poli 1983) that incorporates the Bessel function terms directly:

$$FM_t = \sum_{n=-\infty}^{\infty} J_n(I) \times \sin(2\pi \times [f_c \pm \{n \times f_m\}])t.$$

Each n is an individual partial. So to calculate the amplitude of, say, the third partial, we multiply the third Bessel function at point I, that is, $J_3(I)$, times two sine waves on either side of the carrier frequency. Odd-order lower-side frequency components are phase inverted.

Figure 6.13 depicts the Bessel functions in a three-dimensional representation for n = 1 to 15, with a modulation index range of 0 to 20. The vertical plane (an undulating surface) shows how the amplitudes of the sidebands vary as the modulation index changes. The figure shows that when the number of sidebands is low (at the "back" of the display) the amplitude variation is quite striking. As the number of sidebands increases (shown toward the "front" of the display), the amplitude variations in them (ripples) are small.

From a musical standpoint, the important property is that each Bessel function undulates like a kind of damped sinusoidwide variations for low I and less variation for high I. Simple FM is audibly marked by this indulation as one sweeps the modulation index. Notice also that the $J_n(I)$ for different values of n cross zero at different values of I. So as the modulation index I sweeps, sidebands drop in and out in a quasi-random fashion.

A convenient feature of FM is that the maximum amplitude and signal power do not have to vary with I. This means that as I increases or decreases, the overall amplitude of the tone does not vary wildly. Musically, this means that one can manipulate the amplitude and the index independently by using separate envelopes without worrying about how the value of I will affect the overall amplitude. As we see later in this chapter, this is not the case with some other synthesis techniques, notably waveshaping and the discrete summation formulas. These techniques require *amplitude normalization* since the modulation can drastically affect the output amplitude.

Digital Implementation of FM

Figure 6.9 showed a simple FM instrument in which the depth of modulation is controlled by a constant frequency deviation. But since the bandwidth is directly related to the modulation index and only indirectly to the frequency deviation, it is usually more convenient to specify an FM sound directly in terms of a modulation index. In this case, the instrument needs to be modified to carry out additional calculation according to the following relation:

$D = I \quad M.$

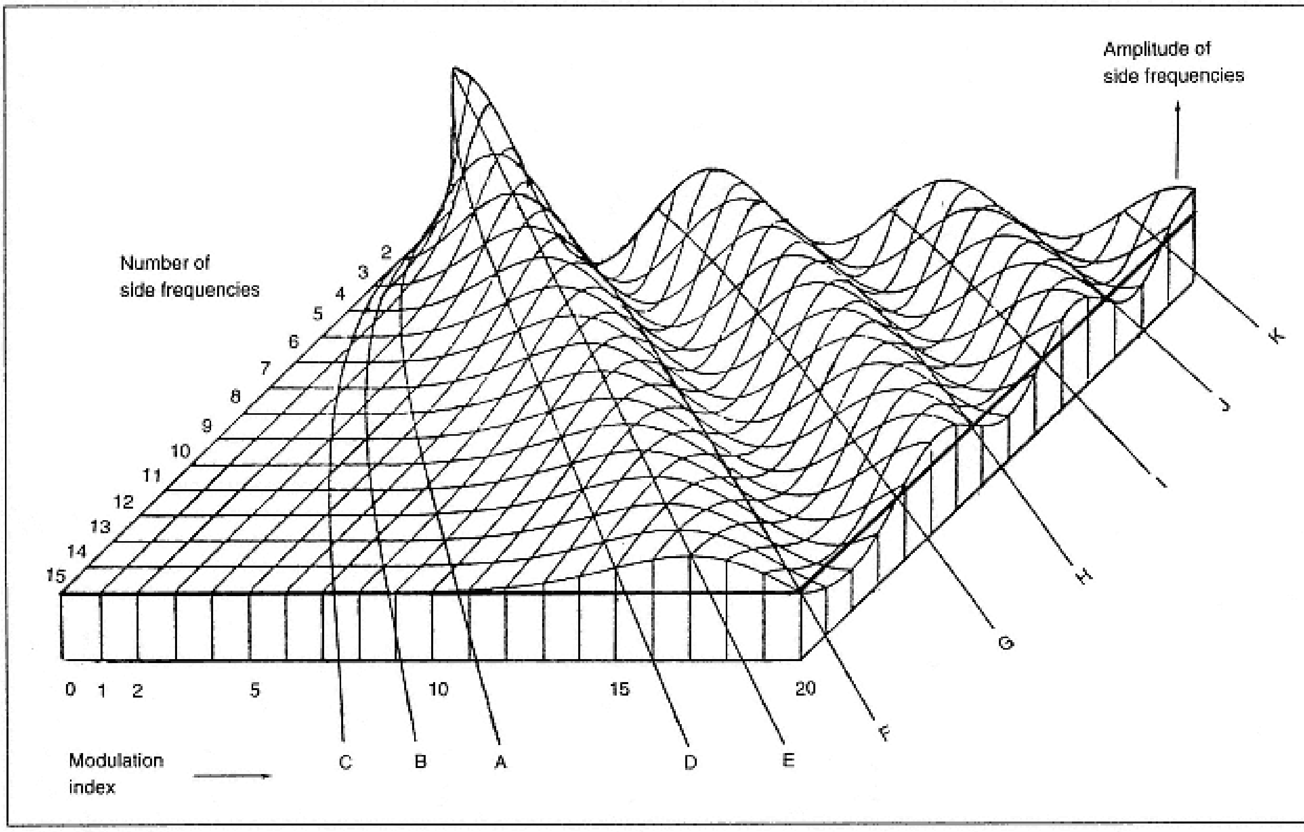

Figure 6.13
Three-dimensional graph of the Bessel functions 1 to 15 plotted (plotted back to front) as a function of modulation (plotted from left to right) showing the number of sidebands generated (after Chowning 1973). Lines A, B, and C points at which the amplitude falls off by 40, 60, and 80 dB, respectively. Line D indicates the cutoff po "perceptually significant" sidebands. E is the maximum amplitude for each order. Lines F through K show the zero of the functions and, therefore, values of the index that produce a null or zero amplitude for various side frequ

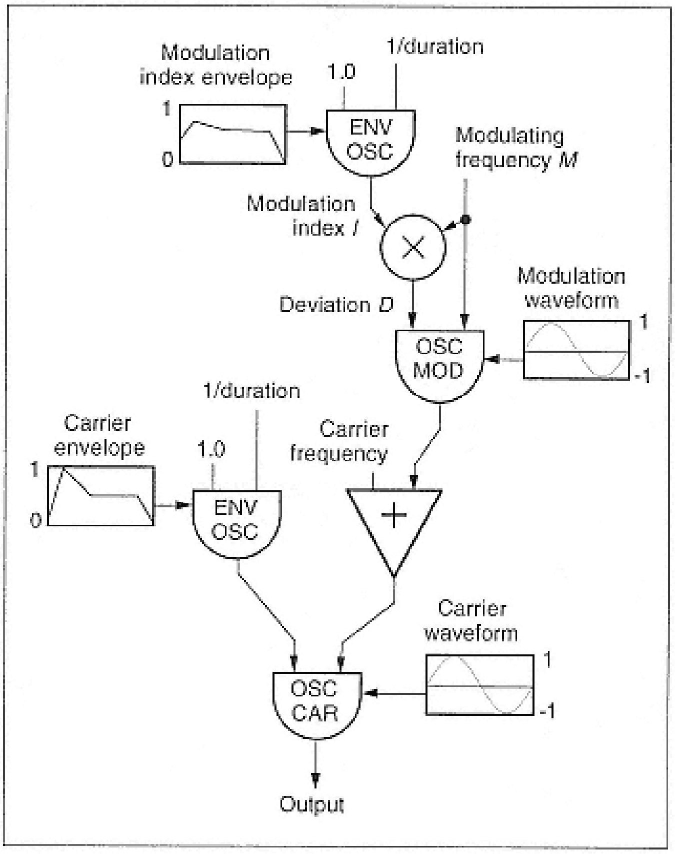

Figure 6.14
Simple FM instrument with envelopes for amplitude and frequency. This instrument also translates a user- specified modulation index envelope into a frequency deviation parameter.

A musician usually wants dynamic control of the overall amplitude as well as the modulation index. Figure 6.14 provides these envelopes. In Chowning's original paper (1973) he described a variation of this instrument with a modulation index that varies between two values *I1* and *I2* according to an envelope. (see Maillard 1976 for another implementation.)

Applications of Simple FM

A straightforward application of simple FM is generating brasslike tones. This family of sounds have a sharp attack on both the amplitude and index envelopes, and maintain a *C:M* ratio of 1. The modulation index should vary between 0 and 7.

When the $C:M$ ratio is 1:2, odd harmonics are generated, making possible a crude clarinet simulation. An irrational $C:M$ ratio like

$$C:\sqrt{2C}$$

yields an inharmonic complex that can simulate percussive and bell-like sounds (Moorer 1977).

Besides simulations of instrumental tones, another way to compose with FM is to take advantage of its "unnatural" properties and the uniquely synthetic spectra it generates. This is the approach taken by composers James Dashow and Barry Truax. Dashow uses FM to "harmonize" (in an extended sense of the word "harmony") pitch dyads (Dashow 1980, 1987; Roads 1985c). Truax systematically mapped out the spectral "families" made possible by various $C:M$ ratios (Truax 1977). For example, certain $C:M$ ratios generate harmonic spectra, while others generate combinations of harmonic and inharmonic spectra. Each $C:M$ ratio is a member of a family of ratios that produce the same spectrum and which vary only in the position of the carrier around which spectral energy is centered. By carefully choosing carrier and modulating frequencies a composer can generate a progression of related timbres with the same set of sidebands.

Another approach to composition with FM is to set a constant C or M and generate a set of related timbres with different $C:M$ ratios.

Exponential FM

In the usual digital implementation of FM, the sidebands are equally spaced around the carrier frequency. We call this *linear FM*. In FM on some analog synthesizers, however, the spacing of sidebands is asymmetrical around the carrier, creating a different type of sound altogether. We call this *exponential FM*. This section explains the difference between these two implementations of FM.

Most analog synthesizers let a voltage-controlled oscillator (VCO) be frequency modulated by another oscillator. However, in order to allow equal-tempered keyboard control of the VCO, the VCO responds to a given voltage in a frequency-dependent way. In particular, a typical VCO responds to a one-volt-per-octave protocol, corresponding to the voltage/octave protocol of analog keyboards. In such a system, for example, the pitch A880 Hz is obtained by applying one more volt to the control input of the VCO than that needed to obtain A440.

In the case of FM, a modulating signal that varies between 1 volt and + 1 volt causes a carrier oscillator set to A440 to vary between A220 and

A880. This means that it modulates 220 Hz downward but 440 Hz upwardan asymmetrical modulation. The average center frequency of the carrier changes, which usually means that the perceived center pitch is detuned by a significant interval. This detuning is caused by the modulation index, which means that the bandwidth and the center frequency are linked. From a musical standpoint, this linkage is not ideal. We want to be able to increase the modulation index without shifting the center frequency. See Hutchins (1975) for an analysis of exponential FM.

In digital modulation the sidebands are spaced equally around the carrier; hence the term *linear FM*. As the modulation index increases, the center frequency remains the same. All digital FM is linear, and at least one manufacturer, Serge Modular, makes a linear FM analog oscillator module.

Analysis and FM

Since FM techniques can create many different families of spectra, it might be useful to have an analysis/resynthesis procedure linked to FM, similar to those used with additive and subtractive techniques. Such a procedure could take an existing sound and translate it into parameter values for an FM instrument. By plugging those values into the instrument, we could hear an approximation of that sound via FM synthesis. The general name for this type of procedure is *parameter estimation* (see chapter 13). Various attempts have been made to try to approximate a given steady-state spectrum automatically using FM (Justice 1979; Risberg 1982). The problem of estimating the FM parameters for complex evolving sounds is difficult (Kronland-Martinet and Grossmann 1991; Horner, Beauchamp, and Haken 1992).

As the power of digital hardware has increased, some of the motivation for estimating FM parameters has diminished. FM synthesis was originally proposed as a computationally efficient method, but now more powerful synthesis methods (such as additive synthesis) are no longer so difficult. Only a certain class of sounds are well modeled as modulations. Additive synthesis and physical models (see chapter 7) may be more appropriate models of traditional instruments.

Multiple-Carrier FM

By *multiple-carrier frequency modulation* (MC FM), we mean an FM instrument in which one oscillator simultaneously modulates two or more carrier oscillators. The output of the carriers sum to a composite waveform that

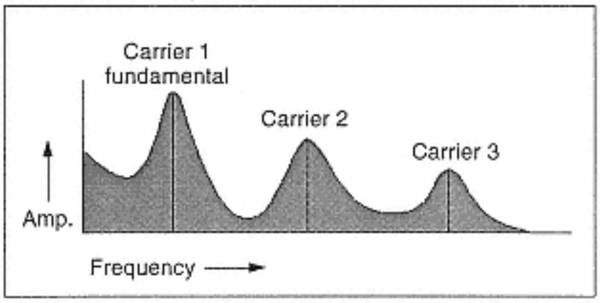

Figure 6.15
A spectrum with three formant regions created with a three-carrier FM instrument.

superposes the modulated spectra. Multiple carriers can create *formant regions* (peaks) in the spectrum, as shown in figure 6.15. The presence of formant regions is characteristic of the spectrum of the human voice and most traditional instruments. Another justification for separate carrier systems is to set different decay times for each formant region. This is useful in simulating brasslike tones where the upper partials decay more rapidly than the lower partials.

Figure 6.16 shows a triple-carrier FM instrument. In order to indicate clearly the multiple-carrier structure, the figure omits envelope controls and waveform tables. The amplitudes of the carriers are independent. When the *Carrier 2* and *Carrier 3* amplitudes are some fraction of *Carrier 1*, the instrument generates formant regions around the frequencies of the second and third carriers.

The equation for a multiple-carrier FM waveform at time *t* is simply the addition of *n* simple FM equations:

$$MCFM_t = A^{w1} \times \sin(C1_t + [I1 \times \sin(M)]) \ldots$$
$$+ A^{wn} \times \sin(Cn_t + [In \times \sin(M)])$$

where *A* is an amplitude constant, $0 < A \Re \leq 1.0$,

w1 is the weighting of *Carrier 1*,

wn is the weighting of *Carrier n*,

C1 is the fundamental pitch = 2π carrier frequency 1 (in Hz),

Cn is the formant frequency = 2π carrier frequency *n* (in Hz), where *Cn* is an integer multiple of *C1*,

M is modulating frequency, usually set to be equal to *C1* (Chowning 1989),

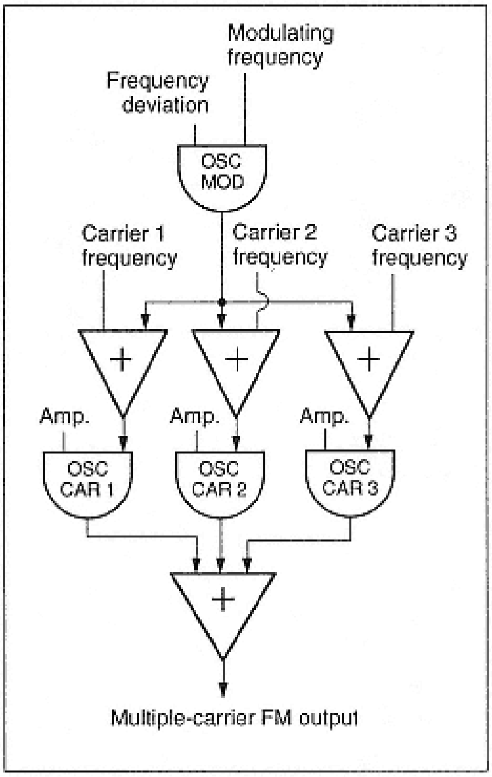

Figure 6.16
Triple-carrier FM instrument driven by a single modulating oscillator (OSC MOD).

I1 is the modulation index of *C1*

In is the modulation index of *Cn*

The exponents *w1* and *wn* determine how the relative contribution of the carriers vary with the overall amplitude *A*.

Musical Applications of MC FM

Documented applications of MC FM strive to simulate the sounds of traditional instrument tones. With MC FMor any synthesis technique, for that matterthe secret of realistic simulation is attention to detail in all aspects of the soundamplitude, frequency, spectral envelopes, vibrato, and musical context.

A straightforward application of MC FM is in the synthesis of trumpet-like tones. Risset and Mathews's (1969) analysis of trumpet-like tones showed a nearly harmonic spectrum, a 20 25 ms rise time of the amplitude envelope (with high partials building up more slowly), a small quasi-random frequency fluctuation, and a formant peak in the region of 1500 Hz. Morrill (1977) developed both single-carrier and double-carrier FM instruments for brass tone synthesis based on these data. A double-carrier instrument sounds more realistic, since each carrier produces frequencies for different parts of the spectrum. In particular, *C1* generates the fundamental and the first five to seven partials, while *C2* is set at 1500 Hz, the main formant region of the trumpet. Each carrier has its own amplitude envelope for adjusting the balance between the two carrier systems in the composite spectrum. For example, in loud trumpet tones, the upper partials standout.

Chowning (1980, 1989) applied the MC FM technique to the synthesis of vowel sounds sung by a soprano and by a low bass voice. He determined that a combination of periodic and random vibrato must be applied to all frequency parameters for realistic simulation of the vocal tones. "Without vibrato the synthesized tones are unnatural sounding" (Chowning 1989, p. 62). A quasi-periodic vibrato makes the frequencies "fuse" into a vocal-like tone. In Chowning's simulations, the *vibrato percent deviation V* is defined by the relation

$$V = 0.2 \quad \log(\text{pitch}).$$

Hence for a pitch of 440 Hz, *V* is about 1.2 percent or 5.3 Hz in depth. The frequency of the vibrato ranges from 5.0 to 6.5 Hz according to the fundamental frequency range of the pitches F3 to F6.

Multiple-Modulator FM

In multiple-modulator frequency modulation (MM FM) more than one oscillator modulates a single carrier oscillator. Two basic configurations are possible: *parallel* and *series* (figure 6.17). MM FM is easiest to understand when the number of modulators is limited to two and their waveforms are sinusoidal.

Parallel MM FM

In parallel MM FM, two sine waves simultaneously modulate a single carrier sine wave. The modulation generates sidebands at frequencies of the form:

$$C \quad (i \quad M1) \quad (k \quad M2)$$

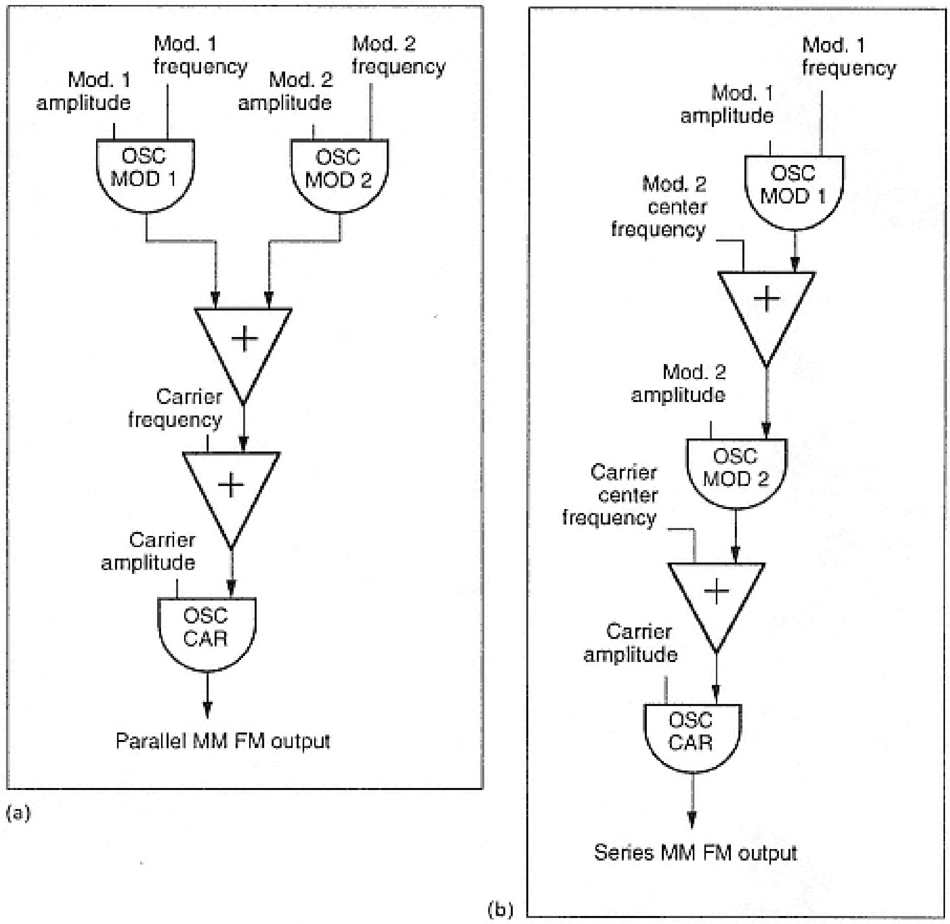

Figure 6.17
MM FM instruments. (*a*) Parallel MM FM. (*b*) Series MM FM.

where *i* and *k* are integers and *M1* and *M2* are the modulating frequencies. In parallel MM FM, it is as though each of the sidebands produced by one of the modulators is modulated as a carrier by the other modulator. The explosion in the number of partials is clear in figure 6.18, which lists both the primary and secondary modulation products.

The wave equation of the parallel double-modulator FM signal at time *t* is as follows:

$$PMMFM_t = A \quad \sin\{C_t + [I1 \quad \sin(M1_t)] + [I2 \quad \sin(M2_t)]\}.$$

For mathematical descriptions of the spectra produced by this class of techniques, see Schottstaedt (1977) and LeBrun (1977).

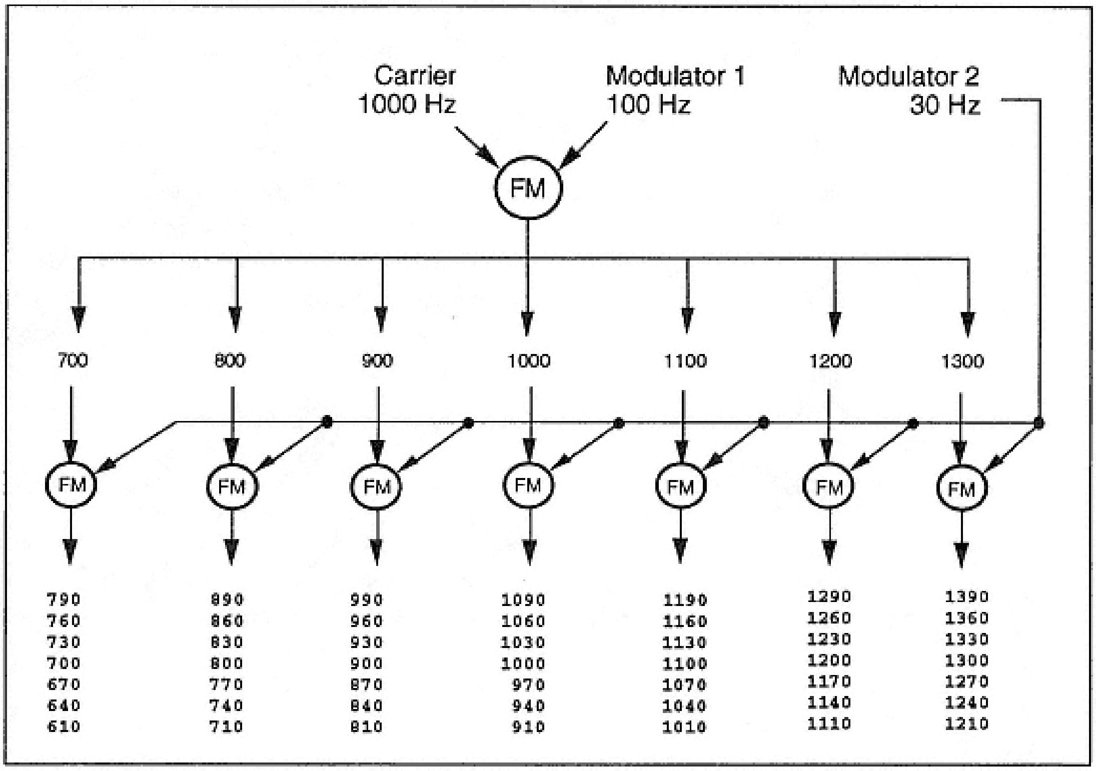

Figure 6.18
This diagram depicts the explosion in the number of partials produced by parallel MM FM. Each of the components emitted by the modulation of the *Carrier* by *Modulator 1* is then modulated by *Modulator 2*, producing the list of spectral components shown at the bottom.

Series MM FM

In series MM FM the modulating sine wave *M1* is itself modulated by *M2*. This creates a complicated modulating wave with a potentially immense number of sinusoidal sideband components, depending on the index of modulation. The instantaneous amplitude of series double-modulator FM is given in the following equation, adapted from Schottstaedt (1977):

$$SMMFM_t = A \quad \sin\{C_t + [I1 \quad \sin(M1_t + [I2 \quad \sin(M2_t)])]\}.$$

The differences between the parallel and serial equations reflects the configuration of the oscillators. In practice, *I2* determines the number of significant sidebands in the modulating signal and *I1* determines the number of sidebands in the output signal. Even small values of *I1* and *I2* create complex waveforms. The ratio *M1:C* determines the placement of the carrier's sidebands, each of which has sidebands of its own at intervals determined by *M2:M1*. Hence, each sideband is modulated and is also a modulator.

Musical Applications of MM FM

Schottstaedt (1977) used double-modulator FM to simulate certain characteristics of piano tones. He set the first modulator to approximately the carrier frequency, and the second modulator to approximately four times the carrier frequency. According to Schottstaedt, if the carrier and the first modulator are exactly equal, the purely harmonic result sounds artificial, like the sound of an electric (amplified tuning bar) piano. This need for inharmonicity in piano tones agrees with the findings of acousticians (Blackham 1965; Backus 1977).

Schottstaedt made the amplitudes of the modulating indexes frequency-dependent. That is, as the carrier frequency increases, the modulation index decreases. The result is a spectrum that is rich in the lower register but becomes steadily simpler as the pitch rises. Since the length of decay of a piano tone also varies with pitch (low tones decay longer), he used a pitch-dependent decay time.

Chowning and Schottstaedt also worked on the simulation of stringlike tones using triple-modulator FM, where the *C:M1:M2* ratio was 1:3:4, and the modulation indexes were frequency dependent (Schottstaedt 1977). Chowning also developed a deep bass voice using a combination MC FM and MM FM instrument. See Chowning (1980, 1989) for more details on this instrument.

Feedback FM

Feedback FM is a widely used synthesis technique, due to Yamaha's patented application of the method in its digital synthesizers (Tomisawa 1981). In this section we describe three types of feedback FM: *one-oscillator feedback, two-oscillator feedback*, and *three-oscillator indirect feedback*.

Feedback FM solves certain problems associated with simple (nonfeedback) FM methods. When the modulation index increases in simple FM, the amplitude of the partials vary unevenly, moving up and down according to the Bessel functions (figure 6.19). This undulation in the amplitude of the partials lends an unnatural "electronic sound" characteristic to the simple FM spectrum; it makes simulations of traditional instruments more difficult.

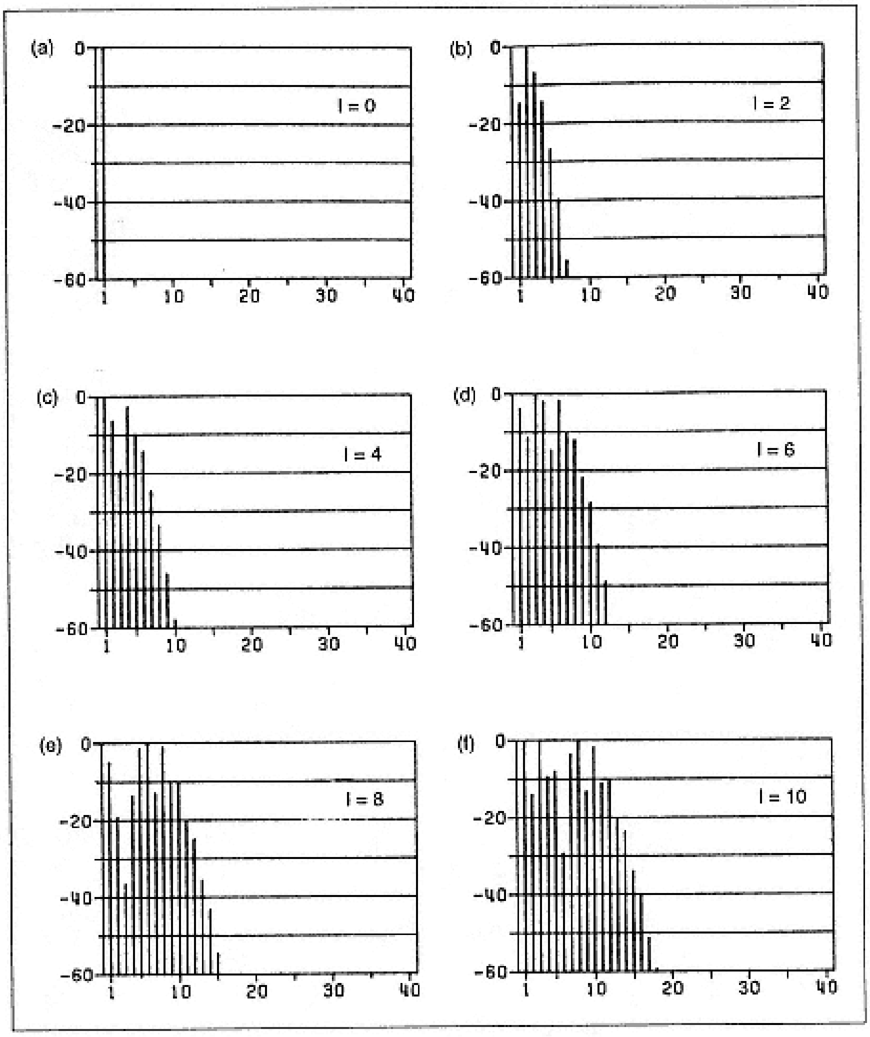

Figure 6.19
A plot of the harmonic spectrum of FM when the frequency of *C* is equal to that of *M*, for values of *I* ranging from 0 to 22 (after Mitsuhashi 1982b). Read the graphs starting from the top left, then top right, then go down a row to the left, then right, etc. Note how uneven the spectrum is, with partials going up and then down as the modulation index changes.

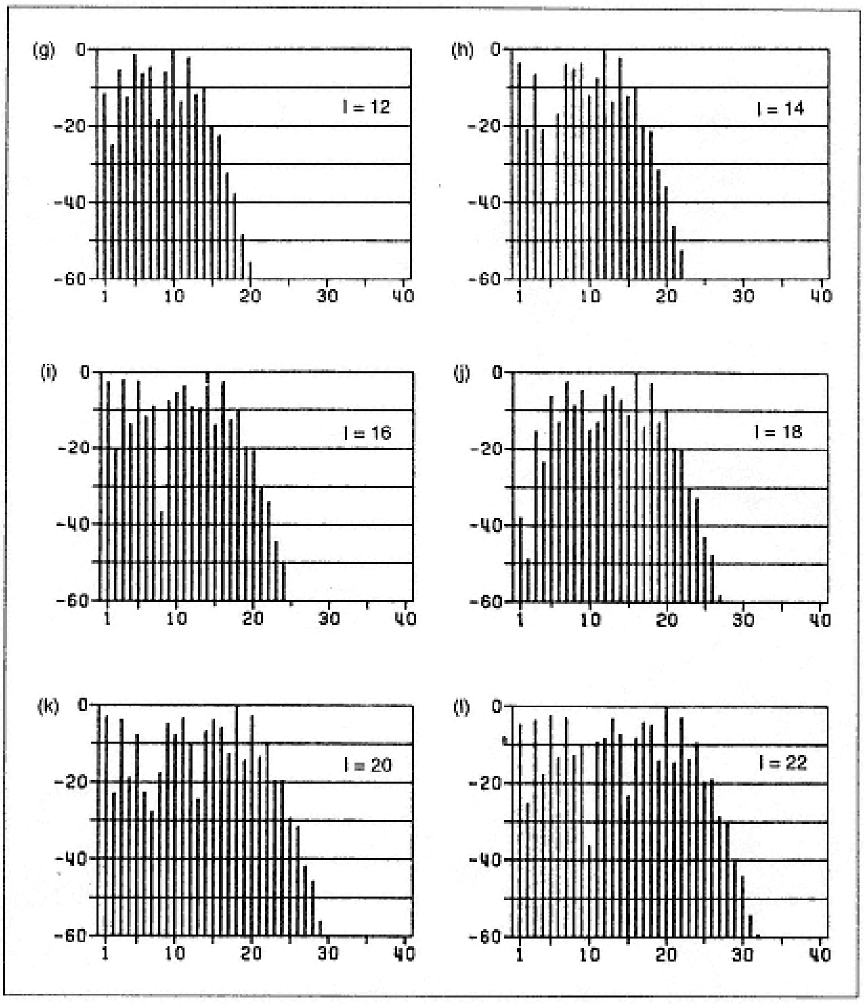

Figure 6.19 (cont.)

Feedback FM makes the spectrum more linear in its evolution. Generally, in feedback FM, as the modulation index increases, the number of partials and their amplitude increases relatively linearly.

Background: Feedback Oscillators

A feedback oscillator instrument first appeared in Jean-Claude Risset's *Introductory Catalog of Computer Generated Sounds* in 1969. Since this catalog was not publicly distributed, the technique first appeared in public in an obscure paper with the cryptic title "Some idiosyncratic aspects of computer synthesized sound" (Layzer 1971). In it, Arthur Layzer described work at Bell Telephone Laboratories in developing a self-modulating oscillator whose output is fed back to its input. This work was a collaboration with Risset, Max Mathews, and F. R. Moore. Moore implemented a feedback oscillator as a unit generator in the Music V language. (Music V is described in Mathews et al. 1969.)

The essential difference between the feedback oscillators developed at Bell Laboratories and the Yamaha feedback FM technique is that the former fed the signal back into the amplitude input, while the latter feeds the signal back into the frequency or phase increment input. Hence the early feedback oscillators were implementing a form of "feedback AM" rather than feedback FM.

One-oscillator Feedback

The basic idea of one-oscillator feedback FM is easy to describe. Figure 6.20 shows an oscillator that feeds its output back into its frequency input through a multiplier and an adder. The adder computes the phase index

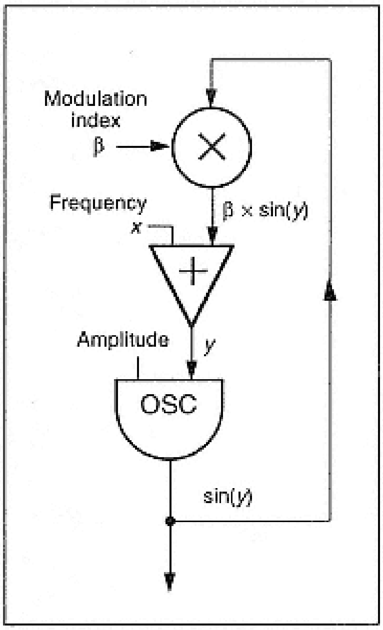

Figure 6.20
Feedback FM instrument. x is a phase increment to a sine wave lookup table. x is added with a signal fed back from the output, multiplied a feedback factor β.

for the sine table-lookup operation within the oscillator. At each sample period, a value x (the frequency increment) is added to the existing phase. The value in the sine table at this new phase is the output signal sin (y). In a synthesizer, x is usually obtained by pressing a key on a musical keyboard. This keystroke translates into a large phase increment value for a high-pitched note or a small phase increment value for a low-pitched note.

In feedback FM, the output signal sin(y) routes back to the adder after being multiplied by the *feedback factor* β. The factor β acts as a kind of scaling function or "modulation index" for the feedback. With the feedback loop the address of the next sample is $x + [\beta \quad \sin(y)]$.

Figure 6.21 plots the spectrum of a one-oscillator feedback FM instrument as β increases. Notice the increase in the number of partials, and the regular, incremental differences in amplitude between the partials, all contributing to a quasi-linear spectral buildup. With increasing modulation, the signal evolves from a sine wave to a sawtooth wave in a continuous manner.

The equation for one-oscillator feedback FM can be characterized by reference to the Bessel functions (Tomisawa 1981):

$$FFM_t = \sum_{n=1}^{\infty} \frac{2}{n \times \beta} \times J_n(n \times \beta) \times \sin(n \times x)t$$

where $J_n(n)$ is a Bessel function of order n and $n \quad \beta$ is the modulation index. The Bessel functions act in different ways in feedback FM as opposed to simple FM. In simple FM, the modulation index I is common for each Bessel component $J_n(I)$. This means that each Bessel function value $J_n(n)$ is represented by a height at a position where the common modulation index crosses. Accordingly, as the modulation index in regular FM increases, the spectral envelope assumes an undulating character. In feedback FM, the order n of the Bessel function $J_n(n \quad \beta)$ is included in the modulation index, and the factor $2/(n \quad \beta)$ is multiplied as a coefficient to the Bessel equation (Mitsuhashi 1982a).

In feedback FM, the modulation index $n \quad \beta$ differs for each order n and increases approximately in the manner of a monotone function (i.e., the increase is by a constant factor). The scaling coefficient $2/n \quad \beta$ ensures that as the order n of partials increases, their amplitude decreases.

Two-oscillator Feedback

Another feedback FM patch takes the output of a feedback oscillator and uses it to modulate another oscillator (figure 6.22). The multiplier M in the figure functions as the index of modulation control between the two oscillators.

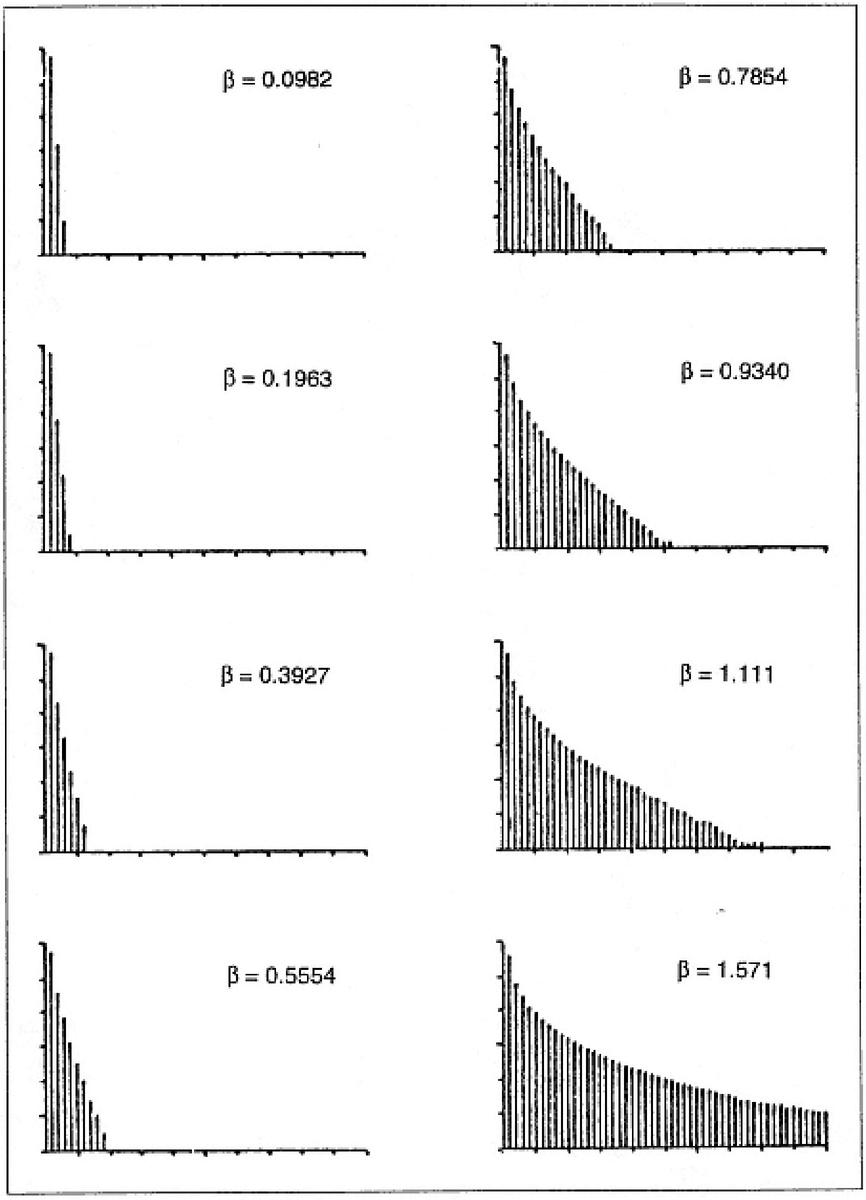

Figure 6.21
Spectrum of a one-oscillator feedback FM instrument as the feedback factor β increases, with the phase increment x set at 200 Hz. The horizontal axis shows frequency plotted from 0 to 10 KHz. The vertical axis shows amplitude on a scale from 0 to 60 dB.

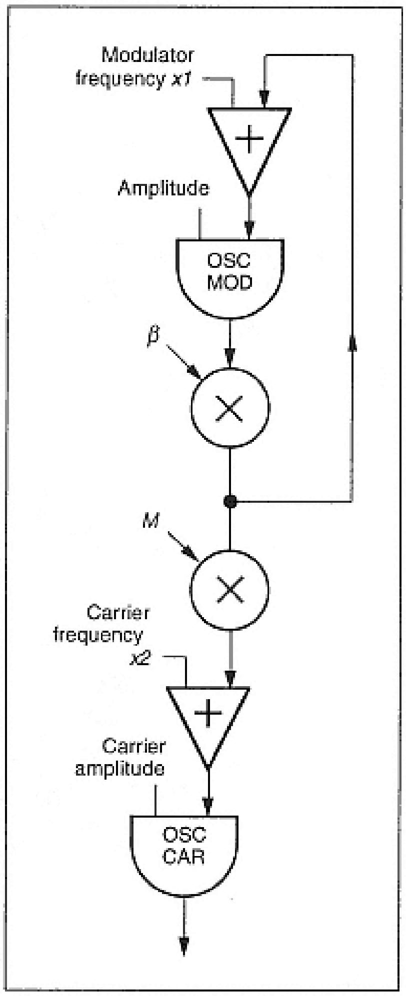

Figure 6.22
Two-oscillator feedback FM instrument. The output of a feedback FM oscillator modulates a second, nonfeedback oscillator.

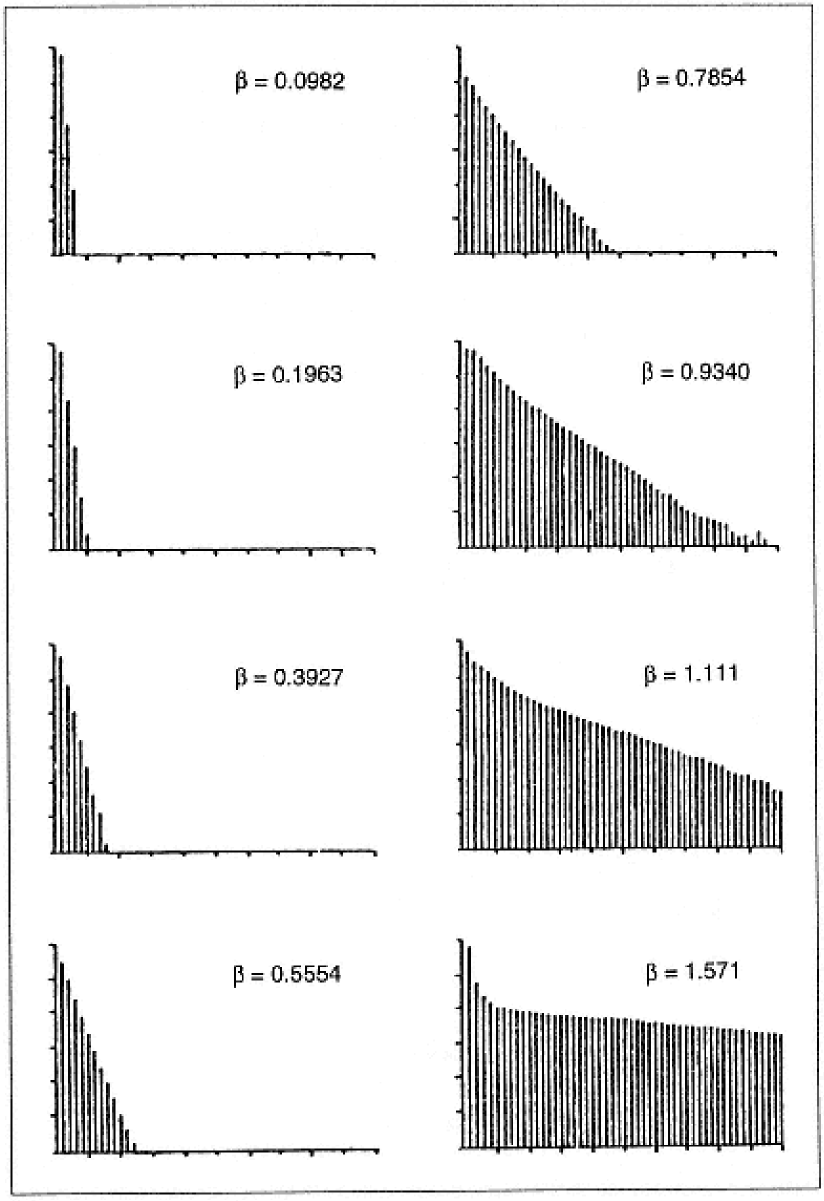

Figure 6.23
Spectrum generated by a two-oscillator feedback FM instrument as the feedback factor β increases from 0.0982 to 1.571. The frequency values for *x1* and *x2* are both set at 200 Hz, and the modulation index *M* is set to the constant value 2. The horizontal axis shows frequency plotted from 0 to 10 KHz. The vertical axis shows amplitude on a scale from 0 to 60 dB.

When M is in the range of 0.5 to 2, the spectrum has a monotonically decreasing tendency in which the amplitude of the partials decreases as the number of partials increases (figure 6.23). When the feedback parameter β is greater than 1, the overall amplitude of the high-order partials increases. This creates the effect of a variable filter. It thus has a more strident and shrill sound. However, when M is set to 1 and $x1$ and $x2$ are equal, this instrument generates the same spectrum as the single-oscillator feedback FM instrument shown in figure 6.20.

When the ratio between $x2$ (the carrier) and $x1$ (the modulator) is 2:1, the modulation index M is 1, and β varies between 0.09 and 1.571, the result is a continuous variation between a quasi-sine wave and a quasi-square wave.

Three-oscillator Indirect Feedback

Another variation on feedback FM in a three-oscillator technique with *indirect feedback*, shown in figure 6.24. The feedback parameter is $\beta 1$. Indirect feedback produces a complex form of modulation. When the frequencies $x1$, $x2$, and $x3$ are noninteger multiples, nonpitched sounds are created. A beating chorus effect is produced when these frequencies are very close to being in an integer relationship. According to sound designer David Bristow (personal communication 1986) this instrument generates a rich spectrum, and when the feedback is increased the energy tends to focus at the high end of the spectrum.

Phase Distortion

Phase distortion (PD) synthesis is a term invented by the Casio corporation to describe a simple modulation technique developed for several of its digital synthesizers. PD synthesis uses a sine wave table-lookup oscillator in which the rate of scanning through the oscillator varies over the cycle. The scanning interval speeds up from 0 to π and then slows down from π to 2π. The overall frequency is constant, according to the pitch of the note, but the output waveform is no longer a sine. Figure 6.25 illustrates the effect of the bent (sped up and then slowed down) scanning function on the output waveform.

As the amount of speeding up and slowing down increases (bending the scanning function progressively), the original sinusoidal waveform turns into a kind of triangle wave, and finally into a quasi-sawtooth waveform that is rich in harmonics.

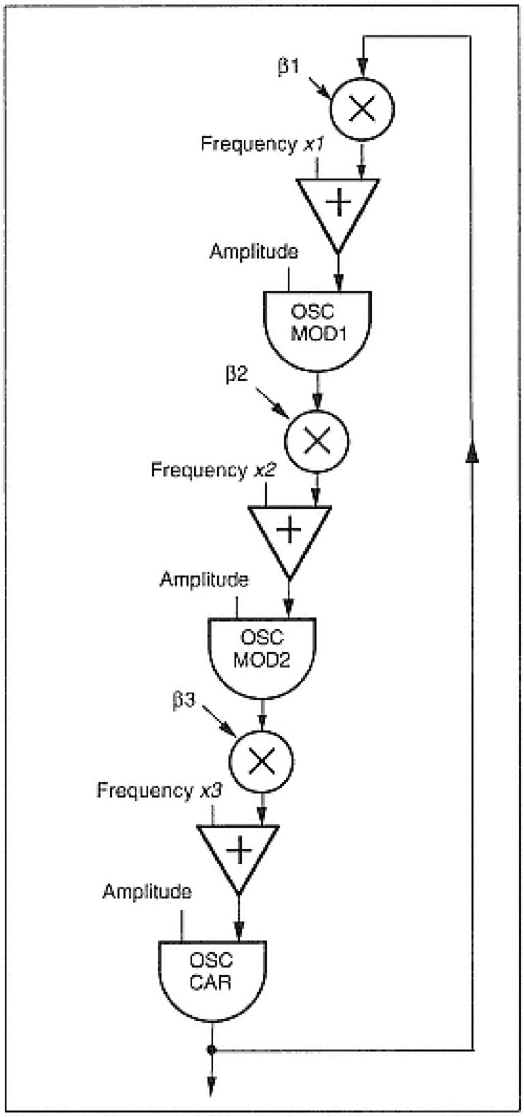

Figure 6.24
Three-oscillator indirect feedback FM instrument. A series of three oscillators modulate each other. Three modulation index factors β*1*, β*2*, and β*3* determine the amount of modulation. The global output is fed back into the first modulating oscillator.