"Modulation" in electronic and computer music means that some aspect of one signal (the *carrier*) varies according to an aspect of a second signal (the *modulator*). The familiar effects of *tremolo* (slow amplitude variation) and *vibrato* (slow frequency variation) in traditional instruments and voices exemplify acoustic modulation. The carrier in these cases is a pitched tone, and the modulator is a relatively slow-varying function (less than 20 Hz). At the right moment and at the right speed, tremolo and vibrato charge both electronic and acoustic tones with expressivity.

When the frequency of modulation rises into the audio bandwidth (above 20 Hz or so), audible *modulation products* or *sidebands* begin to appear. These are new frequencies added to the spectrum of the carrier (typically on either side of the carrier).

To achieve the same complexity of spectrum, modulation synthesis is more efficient in terms of parameter data, memory requirements, and computation time than additive and subtractive synthesis. Modulation uses a small number of oscillators (typically two to six), whereas additive and subtractive techniques need several times this amount of computational power. Modulation is realized by a few table-lookup, multiplication, and addition operations, depending on the type of modulation desired. Because there are fewer parameters than in additive or subtractive techniques, musicians often find modulation techniques easier to manipulate.

By changing parameter values over time, modulation techniques easily produce time-varying spectra. Carefully regulated modulations generate rich dynamic sounds that come close to natural instrumental tones. It is also possible to use modulations in a nonimitative way to venture into the domain of unclassified synthetic sounds.

In this presentation of modulation, we use a minimum of mathematics combined with a liberal dose of instrument diagrams or "patches." These diagrams depict synthesis instruments as a configuration of elementary signal-processing *unit generators*. (See chapter 1 for an introduction to unit generators.)

The modulating signal can vary from a pure sinusoid at a fixed frequency to pure white noise containing all frequencies. See chapter 8 for details on noise modulations.

Bipolar and Unipolar Signals

Two closely related synthesis methods are *ring modulation* and *amplitude modulation* (RM and AM, respectively). In order to comprehend the difference between them, it is important to understand two types of signals that

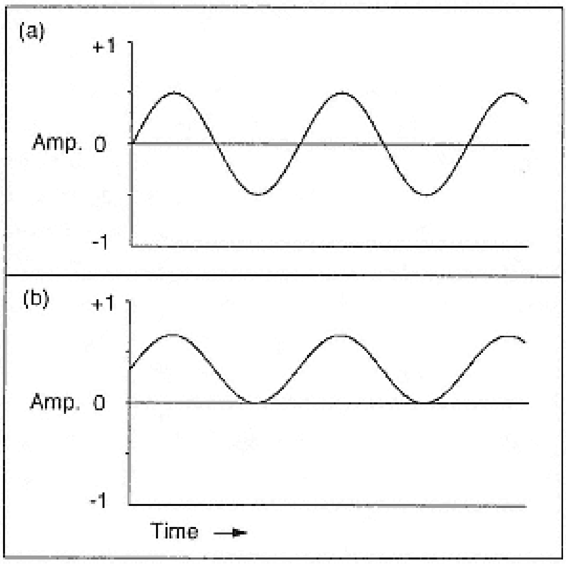

Figure 6.1
Bipolar versus unipolar sine waves. (*a*) Bipolar sine varies between 1 and 1. (*b*) Unipolar sine varies between 0 and 1.

they process: *bipolar* and *unipolar*. A bipolar signal is typical of most audio waveforms, in that it has both a negative and a positive excursion around zero when we look at it in the time domain (figure 6.1a). By contrast, the excursions of a unipolar signal remain within one-half of the full range of the system (figure 6.1b). One way to think of a unipolar signal is that it is a bipolar signal to which a constant has been added. This constant shifts all the sample values to the range above zero. Another term for such a constant is *direct current* (DC) *offset*a signal varying at a frequency of 0 Hz (i.e., not varying).

This distinction is important because the fundamental difference between RM and AM is that RM modulates two bipolar signals, while AM modulates a bipolar signal with a unipolar signal. The next two sections cover both methods in more detail.

Ring Modulation

We start our discussion with RM. In theory, ring modulation is a form of amplitude modulation (Black 1953). In digital systems, RM is simply the multiplication of two bipolar audio signals by one another. That is, a carrier signal *C* is multiplied by a modulator signal *M*. The basic signals *C* and *M* are generated from stored waveforms, and one of them is usually a sine wave. The formula for determining the value of a simple ring-modulated signal *RingMod* at time *t* is a straightforward multiplication:

$RingMod_t = C_t \quad M_t.$

Figure 6.2 portrays two equivalent implementations of at RM instrument. In figure 6.2a it is assumed that the carrier oscillator multiplies the value it reads from the wavetable lookup by the value it takes in from its amplitude input. In figure 6.2b this multiplication is made more explicit. In both cases, the modulator and the carrier vary between 1 and + 1, hence they are bipolar.

When the frequency of the modulator *M* is below 20 Hz or so, the effect of ring modulation is that the amplitude of *C* varies at the frequency of *M*a tremolo effect. But when the frequency of *M* is in the audible range, the timbre of *C* changes. For each sinusoidal component in the carrier, the modulator contributes a pair of *sidebands* to the final spectrum. Given two sine waves as input, RM generates a spectrum that contains two sidebands. These sidebands are the sum and the difference of the frequencies *C* and *M*. Curiously, the carrier frequency itself disappears. Furthermore, if *C* and *M* are in an integer ratio to one another, then the sidebands generated by RM are harmonic; otherwise they are inharmonic.

The sidebands in signal multiplication derive from a standard trigonometric identity:

$\cos(C) \quad \cos(M) = 0.5 \quad [\cos(C \quad M) + \cos(C + M)].$

Yet another way to understand ring modulation is to consider it as a case of *convolution*, as explained in chapter 10.

To give an example of RM, assume that *C* is a 1000 Hz sine wave and *M* is a 400 Hz sine wave. As figure 6.3 shows, their RM spectrum contains a components at 1400 Hz (the sum of *C* and *M*) and 600 Hz (the difference between *C* and *M*).

The phases of the output signal components are also the sum and difference of the phases of the two inputs. If *C* and *M* are more complex signals than sine waves, or if their frequency changes in time, the resulting output spectrum contains many sum and difference frequencies. A spectral plot would show many lines, indicating a complicated spectrum.

Negative Frequencies

As figure 6.3b shows, when the modulating frequency is higher than the carrier frequency, *negative frequencies* occur, as in the case of C = 100 Hz and M = 400 Hz, since $C + M$ = 500, while $C \quad M$ = 300. In spectral

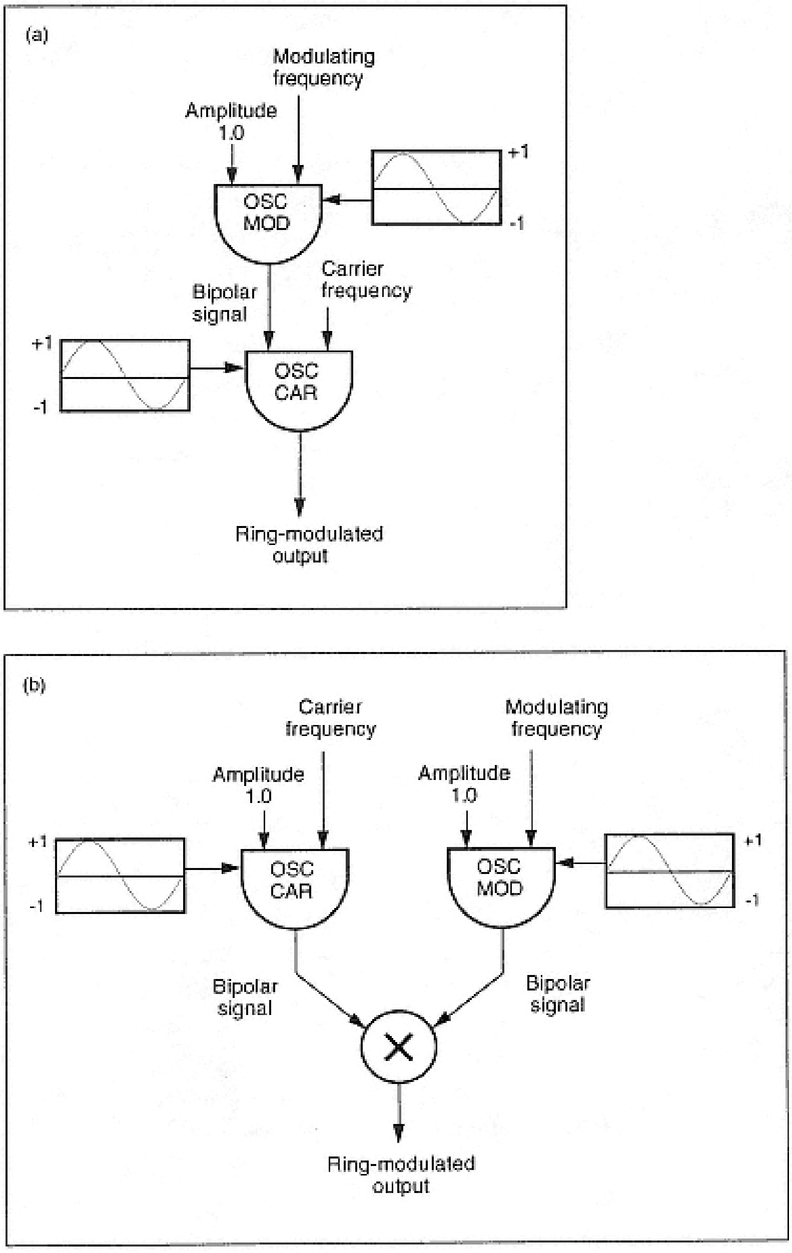

Figure 6.2
Two equivalent implementations of ring modulation or bipolar signal multiplication. The box to the left of each oscillator is its waveform. The top left input of each oscillator is the amplitude, and the top- right input is the frequency. (*a*) RM by implicit multiplication within the carrier oscillator. (*b*) RM by explicit multiplication of the carrier and the modulator signals.

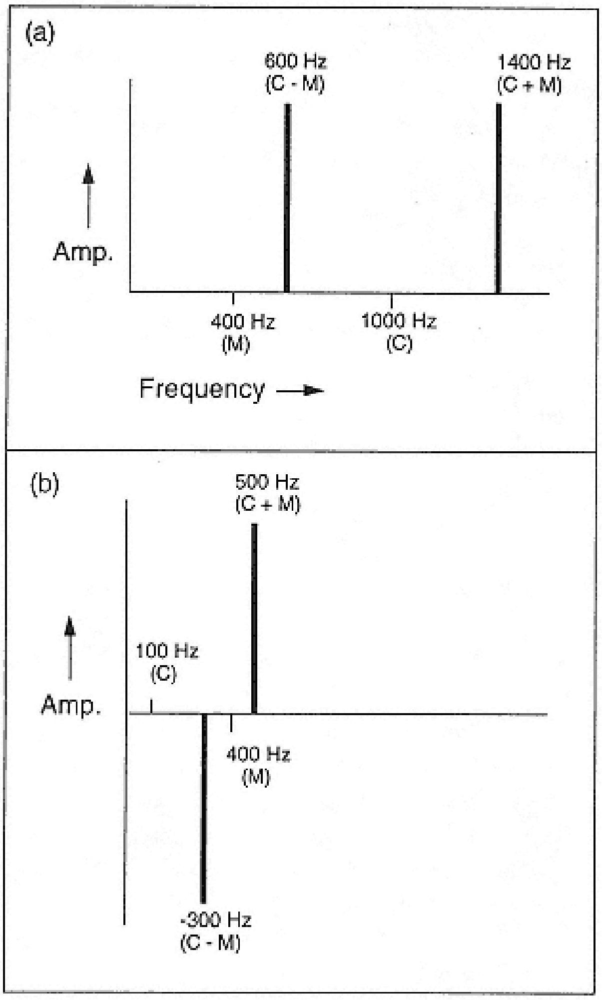

Figure 6.3
Ring modulation spectra. (*a*) For a carrier of 1000 Hz and a modulator of 400 Hz, the sum and difference frequencies are 1400 and 600 Hz, respectively. (*b*) For a carrier of 100 Hz and a modulator of 400 Hz, the sum and difference frequencies are 500 and 300 Hz, respectively.

plots, a negative frequency can be shown as a line extending down from the *x*-axis. The change in sign merely changes the sign of the phase of the signal. (When the sign changes, the waveform flips over the zero or *x*-axis.) Phase becomes important when summing components of identical frequencies, since out-of-phase components can attenuate or cancel in-phase components.

Applications of RM

Typical musical use of RM involves the modification of sampled carrier signals (i.e., the human voice, piano, etc.) by sine wave modulators. Another strategy is to create pure synthetic sounds starting from sine waves in either harmonic or inharmonic ratios. This is the approach taken by composer James Dashow in his pieces such as *Sequence Symbols* (Dashow 1987).

Analog Ring Modulation and Frequency Shifting

Digital ring modulation relies on signal multiplication. In general, digital RM should always sound the same. In contrast, various analog RM circuits have a different "character," depending on the exact circuit and components used. This is because implementations of analog RM approximate pure multiplication with a four-diode circuit arranged in a "ring" configuration. Depending on the type of diodes (silicon or germanium) these circuits introduce extraneous frequencies (Bode 1967, 1984; Stockhausen 1968; Duesenberry 1990; Strange 1983; Wells 1981). For example, in an analog ring modulator based on silicon diodes, the diodes in the circuit clip the carrier (turning it into a quasi-square wave) when it reaches the momentary level of the modulator. This creates the effect of several sums and differences on odd harmonics of the carrier, of the form

$$C + M,\ C \quad M,\ 3C + M,\ 3C \quad M,\ 5C + M,\ 5C \quad M,\ \ldots.$$

Figure 6.4 compares the signals emitted by multiplying RM and diodeclipping RM. Analog ring modulation was used extensively in the electronic music studios of the 1950s, 1960s, and 1970s. The German composer Karlheinz Stockhausen was especially fond of ring modulation; he used it in a number of pieces composed in the 1960s, including *Kontakte, Mikrophonie I* and *II, Telemusik, Hymnen, Prozession*, and *Kurzwellen* (Stockhausen 1968, 1971b).

A pioneer of musical ring modulation, the inventor Harald Bode also developed a variation on RM called *frequency shifting* (Bode 1967, 1984;

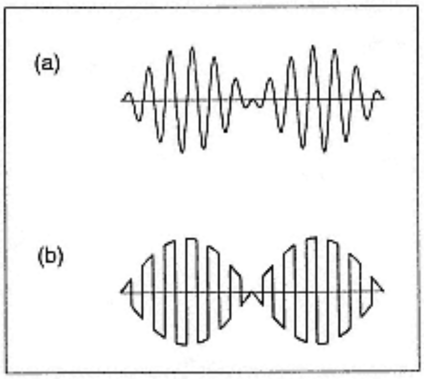

Figure 6.4
Two forms of ring modulation. (*a*) Multiplication RM. (*b*) Diodeclipping or "chopper" RM.

Bode and Moog 1972). A frequency shifter or *Klangumwandler* has separate outputs for the sum and difference frequencies. Another term for this method is *single-sideband modulation* (Oppenheim and Willsky 1983).

Amplitude Modulation

Amplitude modulation is one of the oldest modulation techniques (Black 1953) and has been used extensively in analog electronic music. As in RM, the amplitude of a carrier wave varies in accordance with a modulator wave. The difference between the two techniques is that in AM the modulator is unipolar (the entire waveform is above zero).

Perhaps the most mundane example of infraaudio AM occurs when superposing an envelope onto a sine wave. The envelope, which is unipolar since it varies between 0 and 1, acts as a modulator. The sine wave, which is bipolar since it varies between 1 and + 1, acts as a carrier. To apply an envelope to a signal is to multiply the two waveforms C and M:

$$AmpMod_t = C_t \quad M_t$$

where $AmpMod_t$ the value of an amplitude-modulated signal at time t. Figure 6.5 depicts the result.

Like RM, AM generates a pair of sidebands for every sinusoidal component in the carrier and the modulator. The sidebands are separated from the carrier by a distance corresponding to the inverse of the period of the modulator. The sonic difference between RM and AM is that the AM spectrum contains the carrier frequency as well (figure 6.6). The amplitude of the two

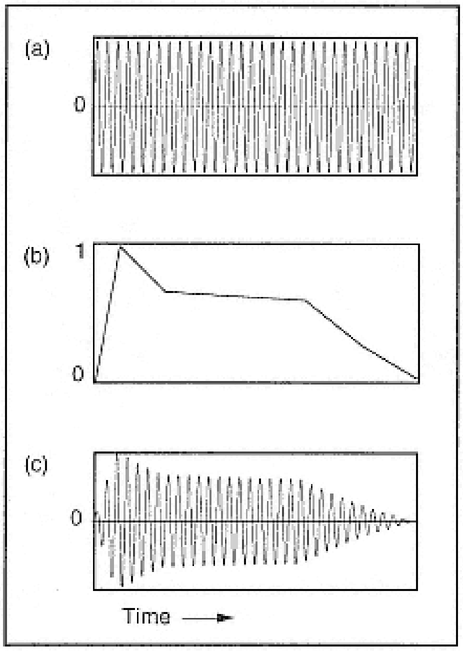

Figure 6.5
Applying an envelope to a signal is a simple case of infra-audio AM. The sine wave signal in (*a*) is multiplied by the envelope signal in (*b*) to produce the enveloped signal in (*c*).

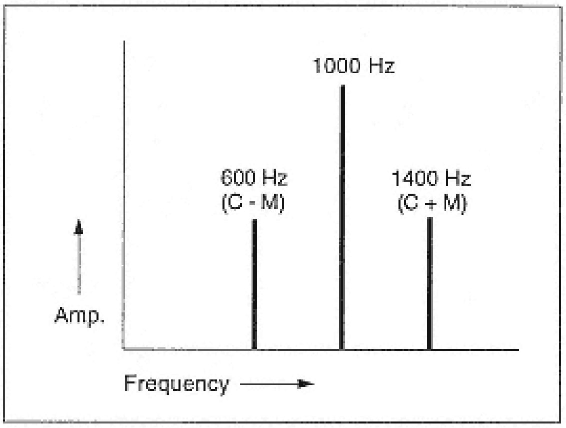

Figure 6.6
Spectrum produced by AM of a 1KHz sine wave by a 400 Hz sine wave. The two sidebands are at sum and difference frequencies around the carrier frequency. The amplitude of the each of the sidebands is *index*/2.

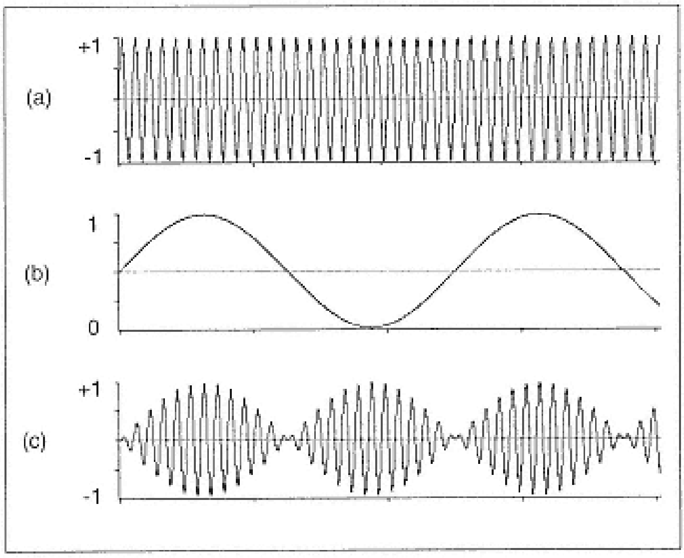

Figure 6.7
Time-domain view of audio frequency AM. The 1 KHz sine wave signal in (*a*) is modulated by the 40 Hz sine wave signal in (*b*) to produced the amplitude modulated signal in (*c*).

sidebands increases in proportion to the amount of modulation, but never exceeds half the level of the carrier.

Figure 6.7 shows a time-domain view of AM created by the modulation of two sine wave signals in the audio band.

AM Instruments

To implement classic AM one restricts the modulator to a unipolar signalthe positive range between 0 and 1. Figure 6.8a shows a simple instrument for AM where the modulator is a unipolar signal.

Modulation Index

A slightly more complicated instrument is needed to control the amount of modulation and the overall amplitude envelope. Figure 6.8b depicts an AM instrument that controls the amount of modulation with an envelope (top left of figure). This envelope functions as a *modulation index*, in the parlance of modulation theory (more on this later). The instrument scales a bipolar modulation signal into a unipolar signal varying between 0 and 1, and then adds this to an overall amplitude envelope over the duration of a sound event. The following equation describes the resulting AM waveform:

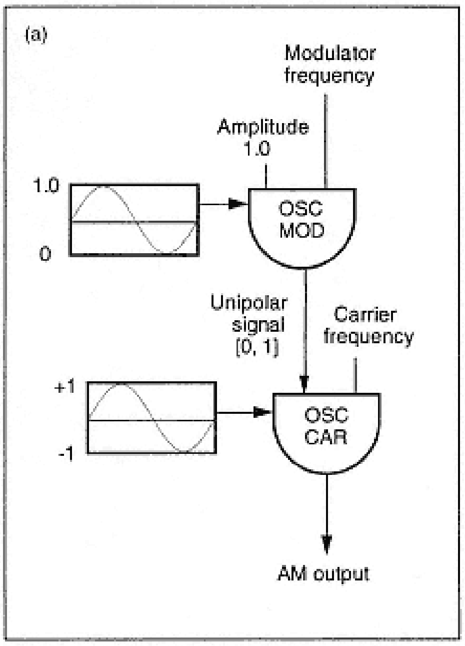

Figure 6.8
Two implementations of AM. (*a*) A simple instrument for AM where the modulating signal is assumed to be unipolar. (*b*) A more complicated instrument for AM with controls for the amount of modulation and the overall amplitude over the duration of the note event. The box to the left of each oscillator is its waveform. In the case of the envelope oscillators (denoted by ENV OSC), the frequency period is 1/*note_duration*. This means that they read through their wavetable once over the duration of a note event. The Positive scaler module ensures that the modulation input to the adder varies between 0 and 0.5.

$$AmpMod = A_c \times \cos(C) + (I \times A_c)/2 \times \cos(C + M) + (I \times A_c)/2 \times \cos(C - M)$$

where *AmpMod* is the amplitude-modulated signal, A_c is the amplitude of the carrier, *I* is the modulation index, *C* is the carrier frequency, and *M* is the modulator frequency.

Frequency Modulation

Frequency modulation (FM) is a very well known digital synthesis method, due to its adoption by the Yamaha corporation. However, FM is not one technique, but a family of methods that share the common property of wavetable lookup according to a nonlinear oscillating function.